

## SECTION 110

### METHODS OF ESTIMATING PERCENTAGE OF MATERIAL WITHIN SPECIFICATIONS LIMITS (PWL)

**110-01 GENERAL.** When the specifications provide for acceptance of material based on the method of estimating percentage of material within specification limits (PWL), the PWL will be determined in accordance with this section. All test results for a lot will be analyzed statistically to determine the total estimated percent of the lot that is within specification limits. The PWL is computed using the sample average ( $\bar{X}$ ) and sample standard deviation ( $S_n$ ) of the specified number ( $n$ ) of sublots for the lot and the specification tolerance limits,  $L$  for lower and  $U$  for upper, for the particular acceptance parameter. From these values, the respective Quality index(s),  $Q_L$  for Lower Quality Index and/or  $Q_U$  for Upper Quality Index, is computed and the PWL for the lot for the specified  $n$  is determined from Table 1.

There is some degree of uncertainty (risk) in the measurement for acceptance because only a fraction of production material (the population) is sampled and tested. This uncertainty exists because all portions of the production material have the same probability to be randomly sampled. The Contractor's risk is the probability that material produced at the acceptable quality level is rejected or subjected to a pay adjustment. TxDOT's risk is the probability that material produced at the rejectable quality level is accepted.

IT IS THE INTENT OF THIS SECTION TO INFORM THE CONTRACTOR THAT, IN ORDER TO CONSISTENTLY OFFSET THE CONTRACTOR'S RISK FOR MATERIAL EVALUATED, PRODUCTION QUALITY (USING POPULATION AVERAGE AND POPULATION STANDARD DEVIATION) MUST BE MAINTAINED AT THE ACCEPTABLE QUALITY SPECIFIED OR HIGHER. IN ALL CASES, IT IS THE RESPONSIBILITY OF THE CONTRACTOR TO PRODUCE AT QUALITY LEVELS THAT WILL MEET THE SPECIFIED ACCEPTANCE CRITERIA WHEN SAMPLED AND TESTED AT THE FREQUENCIES SPECIFIED.

**110-02 METHOD FOR COMPUTING PWL.** The computational sequence for computing PWL is as follows:

- a. Divide the lot into  $n$  sublots in accordance with the acceptance requirements of the specification.
- b. Locate the random sampling position within the subplot in accordance with the requirements of the specification.
- c. Make a measurement at each location, or take a test portion and make the measurement on the test portion in accordance with the testing requirements of the specification.
- d. Find the sample average ( $\bar{X}$ ) for all subplot values within the lot by using the following formula:

$$\bar{X} = (x_1 + x_2 + x_3 + \dots + x_n) / n$$

Where:

$\bar{X}$  = Sample average of all subplot values within a lot  
 $x_1, x_2$  = Individual subplot values  
 $n$  = Number of sublots

- e. Find the sample standard deviation ( $S_n$ ) by use of the following formula:

$$S_n = [(d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2) / (n-1)]^{1/2}$$

Where:

$S_n$  = Sample standard deviation of the number of subplot values in the set

$d_1, d_2$  = Deviations of the individual subplot values  $X_1, X_2$  .  
 . from the average value  $\bar{X}$

that is:  $d_1 = (x_1 - \bar{X}), d_2 = (x_2 - \bar{X}) \dots d_n = (x_n - \bar{X})$

$n$  = Number of sublots

- f. For single sided specification limits (i.e., L only), compute the Lower Quality Index  $Q_L$  by use of the following formula:

$$Q_L = (\bar{X} - L) / S_n$$

Where:

L = specification lower tolerance limit

Estimate the percentage of material within limits (PWL) by entering Table 1 with  $Q_L$ , using the column appropriate to the total number ( $n$ ) of measurements. If the value of  $Q_L$  falls between values shown on the table, use the next higher value of PWL.

- g. For double sided specification limits (i.e. L and U), compute the Quality Indexes  $Q_L$  and  $Q_U$  by use of the following formulas:

$$Q_L = (\bar{X} - L) / S_n \quad \text{and} \quad Q_U = (U - \bar{X}) / S_n$$

Where:

L and U = specification lower and upper tolerance limits

Estimate the percentage of material between the lower (L) and upper (U) tolerance limits (PWL) by entering Table 1 separately with  $Q_L$  and  $Q_U$ , using the column appropriate to the total number ( $n$ ) of measurements, and determining the percent of material above  $P_L$  and percent of material below  $P_U$  for each tolerance limit. If the values of  $Q_L$  fall between values shown on the table, use the next higher value of  $P_L$  or  $P_U$ . Determine the PWL by use of the following formula:

$$PWL = (P_U + P_L) - 100$$

Where:

$P_L$  = percent within lower specification limit

$P_U$  = percent within upper specification limit

### EXAMPLE OF PWL CALCULATION

**Project:** Example Project  
**Test Item:** Item P-401, Lot A.

#### A. PWL Determination for Mat Density.

1. Density of four random cores taken from Lot A.

A-1	96.60
A-2	97.55
A-3	99.30
A-4	98.35

$n = 4$

2. Calculate average density for the lot.

$$X = (x_1 + x_2 + x_3 + \dots + x_n) / n$$
$$X = (96.60 + 97.55 + 99.30 + 98.35) / 4$$
$$X = 97.95 \text{ percent density}$$

3. Calculate the standard deviation for the lot.

$$S_n = [((96.60 - 97.95)^2 + (97.55 - 97.95)^2 + (99.30 - 97.95)^2 + (98.35 - 97.95)^2) / (4 - 1)]^{1/2}$$
$$S_n = [(1.82 + 0.16 + 1.82 + 0.16) / 3]^{1/2}$$
$$S_n = 1.15$$

4. Calculate the Lower Quality Index  $Q_L$  for the lot. ( $L=96.3$ )

$$Q_L = (X - L) / S_n$$
$$Q_L = (97.95 - 96.30) / 1.15$$
$$Q_L = 1.4384$$

5. Determine PWL by entering Table 1 with  $Q_L=1.44$  and  $n=4$ .

PWL = 98

#### B. PWL Determination for Air Voids.

1. Air Voids of four random samples taken from Lot A.

A-1	5.00
A-2	3.74

A-3            2.30  
A-4            3.25

2. Calculate the average air voids for the lot.

$$\begin{aligned} X &= (x_1 + x_2 + x_3 \dots n) / n \\ X &= (5.00 + 3.74 + 2.30 + 3.25) / 4 \\ X &= 3.57 \text{ percent} \end{aligned}$$

3. Calculate the standard deviation  $S_n$  for the lot.

$$\begin{aligned} S_n &= [((3.57 - 5.00)^2 + (3.57 - 3.74)^2 + (3.57 - 2.30)^2 + (3.57 - 3.25)^2) / (4 - 1)]^{1/2} \\ S_n &= [(2.04 + 0.03 + 1.62 + 0.10) / 3]^{1/2} \\ S_n &= 1.12 \end{aligned}$$

4. Calculate the Lower Quality Index  $Q_L$  for the lot. ( $L = 2.0$ )

$$\begin{aligned} Q_L &= (X - L) / S_n \\ Q_L &= (3.57 - 2.00) / 1.12 \\ Q_L &= 1.3992 \end{aligned}$$

5. Determine  $P_L$  by entering Table 1 with  $Q_L = 1.40$  and  $n = 4$ .

$$P_L = 97$$

6. Calculate the Upper Quality Index  $Q_U$  for the lot. ( $U = 5.0$ )

$$\begin{aligned} Q_U &= (U - X) / S_n \\ Q_U &= (5.00 - 3.57) / 1.12 \\ Q_U &= 1.2702 \end{aligned}$$

7. Determine  $P_U$  by entering Table 1 with  $Q_U = 1.27$  and  $n = 4$ .

$$P_U = 93$$

8. Calculate Air Voids PWL

$$\begin{aligned} PWL &= (P_L + P_U) - 100 \\ PWL &= (97 + 93) - 100 = 90 \end{aligned}$$

**TABLE 1. TABLE FOR ESTIMATING PERCENT OF LOT WITHIN LIMITS (PWL)**

Percent Within Limits ( $P_L$ and $P_U$ )	Positive Values of Q ( $Q_L$ and $Q_U$ ) Limits					
	n=3	n=4	n=5	n=6	n=7	n=8
99	1.1541	1.4700	1.6714	1.8008	1.8888	1.9520
98	1.1524	1.4400	1.6016	1.6982	1.7612	1.8053
97	1.1496	1.4100	1.5427	1.6181	1.6661	1.6993
96	1.1456	1.3800	1.4897	1.5497	1.5871	1.6127
95	1.1405	1.3500	1.4407	1.4887	1.5181	1.5381
94	1.1342	1.3200	1.3946	1.4329	1.4561	1.4716
93	1.1269	1.2900	1.3508	1.3810	1.3991	1.4112
92	1.1184	1.2600	1.3088	1.3323	1.3461	1.3554
91	1.1089	1.2300	1.2683	1.2860	1.2964	1.3032
90	1.0982	1.2000	1.2290	1.2419	1.2492	1.2541
89	1.0864	1.1700	1.1909	1.1995	1.2043	1.2075
88	1.0736	1.1400	1.1537	1.1587	1.1613	1.1630
87	1.0597	1.1100	1.1173	1.1191	1.1199	1.1204
86	1.0448	1.0800	1.0817	1.0808	1.0800	1.0794
85	1.0288	1.0500	1.0467	1.0435	1.0413	1.0399
84	1.0119	1.0200	1.0124	1.0071	1.0037	1.0015
83	0.9939	0.9900	0.9785	0.9715	0.9672	0.9643
82	0.9749	0.9600	0.9452	0.9367	0.9325	0.9281
81	0.9550	0.9300	0.9123	0.9025	0.8966	0.8928
80	0.9342	0.9000	0.8799	0.8690	0.8625	0.8583
79	0.9124	0.8700	0.8478	0.8360	0.8291	0.8245
78	0.8897	0.8400	0.8160	0.8036	0.7962	0.7915
77	0.8662	0.8100	0.7846	0.7716	0.7640	0.7590
76	0.8417	0.7800	0.7535	0.7401	0.7322	0.7271
75	0.8165	0.7500	0.7226	0.7089	0.7009	0.6958
74	0.7904	0.7200	0.6921	0.6781	0.6701	0.6649
73	0.7636	0.6900	0.6617	0.6477	0.6396	0.6344
72	0.7360	0.6600	0.6316	0.6176	0.6095	0.6044
71	0.7077	0.6300	0.6016	0.5878	0.5798	0.5747
70	0.6787	0.6000	0.5719	0.5583	0.5504	0.5454
69	0.6490	0.5700	0.5423	0.5290	0.5213	0.5164
68	0.6187	0.5400	0.5129	0.4999	0.4924	0.4877
67	0.5878	0.5100	0.4836	0.4710	0.4638	0.4592
66	0.5563	0.4800	0.4545	0.4424	0.4354	0.4310
65	0.5242	0.4500	0.4255	0.4139	0.4073	0.4031
64	0.4916	0.4200	0.3967	0.3856	0.3793	0.3753
63	0.4586	0.3900	0.3679	0.3575	0.3515	0.3477
62	0.4251	0.3600	0.3392	0.3295	0.3239	0.3203
61	0.3911	0.3300	0.3107	0.3016	0.2964	0.2931
60	0.3568	0.3000	0.2822	0.2738	0.2691	0.2660
59	0.3222	0.2700	0.2537	0.2461	0.2418	0.2391
58	0.2872	0.2400	0.2254	0.2186	0.2147	0.2122
57	0.2519	0.2100	0.1971	0.1911	0.1877	0.1855
56	0.2164	0.1800	0.1688	0.1636	0.1607	0.1592
55	0.1806	0.1500	0.1408	0.1363	0.1338	0.1322
54	0.1447	0.1200	0.1125	0.1090	0.1070	0.1057
53	0.1087	0.0900	0.0843	0.0817	0.0802	0.0792
52	0.0725	0.0600	0.0562	0.0544	0.0534	0.0528
51	0.0363	0.0300	0.0281	0.0272	0.0267	0.0264
50	0.0	0.0	0.0	0.0	0.0	0.0

**TABLE 1. TABLE FOR ESTIMATING PERCENT OF LOT WITHIN LIMITS (PWL)**

Percent Within Limits (P <sub>L</sub> and P <sub>U</sub> )	Negative Values of Q (Q <sub>L</sub> and Q <sub>U</sub> ) Limits					
	n=3	n=4	n=5	n=6	n=7	n=8
49	-.0363	-.0300	-.0281	-.0272	-.0267	-.0264
48	-.0725	-.0600	-.0562	-.0544	-.0534	-.0528
47	-.1087	-.0900	-.0843	-.0817	-.0802	-.0792
46	-.1447	-.1200	-.1125	-.1090	-.1070	-.1057
45	-.1806	-.1500	-.1408	-.1363	-.1338	-.1322
44	-.2164	-.1800	-.1688	-.1636	-.1607	-.1592
43	-.2519	-.2100	-.1971	-.1911	-.1877	-.1855
42	-.2872	-.2400	-.2254	-.2186	-.2147	-.2122
41	-.3222	-.2700	-.2537	-.2461	-.2418	-.2391
40	-.3568	-.3000	-.2822	-.2738	-.2691	-.2660
39	-.3911	-.3300	-.3107	-.3016	-.2964	-.2931
38	-.4251	-.3600	-.3392	-.3295	-.3239	-.3203
37	-.4586	-.3900	-.3679	-.3575	-.3515	-.3477
36	-.4916	-.4200	-.3967	-.3856	-.3793	-.3753
35	-.5242	-.4500	-.4255	-.4139	-.4073	-.4031
34	-.5563	-.4800	-.4545	-.4424	-.4354	-.4310
33	-.5878	-.5100	-.4836	-.4710	-.4638	-.4592
32	-.6187	-.5400	-.5129	-.4999	-.4924	-.4877
31	-.6490	-.5700	-.5423	-.5290	-.5213	-.5164
30	-.6787	-.6000	-.5719	-.5583	-.5504	-.5454
29	-.7077	-.6300	-.6016	-.5878	-.5798	-.5747
28	-.7360	-.6600	-.6316	-.6176	-.6095	-.6044
27	-.7636	-.6900	-.6617	-.6477	-.6396	-.6344
26	-.7904	-.7200	-.6921	-.6781	-.6701	-.6649
25	-.8165	-.7500	-.7226	-.7089	-.7009	-.6958
24	-.8417	-.7800	-.7535	-.7401	-.7322	-.7271
23	-.8662	-.8100	-.7846	-.7716	-.7640	-.7590
22	-.8897	-.8400	-.8160	-.8036	-.7962	-.7915
21	-.9124	-.8700	-.8478	-.8360	-.8291	-.8245
20	-.9342	-.9000	-.8799	-.8690	-.8625	-.8583
19	-.9550	-.9300	-.9123	-.9025	-.8966	-.8928
18	-.9749	-.9600	-.9452	-.9367	-.9325	-.9281
17	-.9939	-.9900	-.9785	-.9715	-.9672	-.9643
16	-1.0119	-1.0200	-1.0124	-1.0071	-1.0037	-1.0015
15	-1.0288	-1.0500	-1.0467	-1.0435	-1.0413	-1.0399
14	-1.0448	-1.0800	-1.0817	-1.0808	-1.0800	-1.0794
13	-1.0597	-1.1100	-1.1173	-1.1191	-1.1199	-1.1204
12	-1.0736	-1.1400	-1.1537	-1.1587	-1.1613	-1.1630
11	-1.0864	-1.1700	-1.1909	-1.1995	-1.2043	-1.2075
10	-1.0982	-1.2000	-1.2290	-1.2419	-1.2492	-1.2541
9	-1.1089	-1.2300	-1.2683	-1.2860	-1.2964	-1.3032
8	-1.1184	-1.2600	-1.3088	-1.3323	-1.3461	-1.3554
7	-1.1269	-1.2900	-1.3508	-1.3810	-1.3991	-1.4112
6	-1.1342	-1.3200	-1.3946	-1.4329	-1.4561	-1.4716
5	-1.1405	-1.3500	-1.4407	-1.4887	-1.5181	-1.5381
4	-1.1456	-1.3800	-1.4897	-1.5497	-1.5871	-1.6127
3	-1.1496	-1.4100	-1.5427	-1.6181	-1.6661	-1.6993
2	-1.1524	-1.4400	-1.6016	-1.6982	-1.7612	-1.8053
1	-1.1541	-1.4700	-1.6714	-1.8008	-1.8888	-1.9520

**END OF SECTION 110**